(1) Let $f : S' \to S$ be a morphism of schemes. Write $(-)_+ : \mathcal{S}\mathrm{pc}(S) \to \mathcal{S}\mathrm{pc}(S)_*$ for the left adjoint to the forgetful functor, and similarly over S'.
   (a) Construct a natural transformation $(-)_+ f_* \to f_*(-)_+$.
   (b) Show that this natural transformation is not always an equivalence. [*Hint:* consider $f = \nabla : \mathrm{Spec}(k) \amalg \mathrm{Spec}(k) \to \mathrm{Spec}(k)$.]
   (c) Let S be noetherian and f a universal homeomorphism. Show that the natural transformation is an equivalence. [*Hint:* show first that if $X \in \mathrm{Sm}_S$ is connected and $F \in \mathcal{S}\mathrm{pc}(S)$, then $(F_+)(X) \simeq F(X)_+$.]

(2) Let $\mathcal{C}$ be a closed symmetric monoidal ∞-category.
   (a) Show that $E \in \mathcal{C}$ is invertible if and only if and only if the counit $\underline{\mathrm{Hom}}(E, \mathbb{1}) \otimes E \to \mathbb{1}$ is an equivalence.
   (b) Show that $E \in \mathcal{C}$ is dualizable if and only if for all $X \in \mathcal{C}$ (respectively for $X = E$) the canonical map $\underline{\mathrm{Hom}}(E, \mathbb{1}) \otimes E \to \underline{\mathrm{Hom}}(E, E)$ is an equivalence.
   (c) Suppose $\mathcal{C}$ is stable (respectively idempotent complete). Show that the same is true for the full subcategory of dualizable objects.

(3) (a) Let $R : \mathcal{D} \to \mathcal{C}$ be a functor of ∞-categories. Show that there is a maximal full subcategory $\mathcal{C}_0 \subset \mathcal{C}$ and a functor $L : \mathcal{C}_0 \to \mathcal{D}$ which is "partially left adjoint to R". (Make sense of this concept.)
   (b) Consider the category Pr^L_m of pairs $(\mathcal{C}, S)$ where $\mathcal{C}$ is a presentable ∞-category and S is a class of morphisms in $\mathcal{C}$ which is strongly saturated and of small generation. Show that there is a functor $Pr^L_m \to Pr^L$ which sends $(\mathcal{C}, S)$ to $\mathcal{C}[S^{-1}]$.
   (c) Show that there is a functor $\mathcal{SH} : \mathrm{Sch}^{\mathrm{op}} \to \mathcal{C}\mathrm{at}$ (sending X to the motivic stable category on X.)

(4) Let S be a scheme and S'/S étale. Show that $\mathcal{S}\mathrm{hv}_{\mathrm{Nis}}(\mathrm{Sm}_S)_{/S'} \simeq \mathcal{S}\mathrm{hv}_{\mathrm{Nis}}(\mathrm{Sm}_{S'})$.