

- (1) Let A be a strictly \mathbb{A}^1 -invariant sheaf of abelian groups on Sm_k , viewed as an object of $\mathcal{SH}^{S^1}(k)^\heartsuit$. Show that $\Omega_{\mathbb{G}_m}(A) \in \mathcal{SH}^{S^1}(k)$ corresponds to the sheaf A_{-1} (with $A_{-1}(X)$ defined as a summand of $A(X \times \mathbb{G}_m)$).
- (2) Let $X \in \mathcal{Spc}(k)_*$. Show that $\Sigma^\infty X \in \mathcal{SH}(k)_{\geq 0}$.
- (3) Show that the homotopy t -structure on $\mathcal{SH}(k)$ is left complete, i.e., every object is the limit of its Postnikov tower. Define a dual notion of right completeness and show that $\mathcal{SH}(k)$ is also right complete.
- (4*) Write $\mathcal{SH}(k)^{\mathrm{eff}} \subset \mathcal{SH}(k)$ for the subcategory generated under colimits by the image of $\mathcal{SH}^{S^1}(k) \rightarrow \mathcal{SH}(k)$. Construct a t -structure on $\mathcal{SH}(k)^{\mathrm{eff}}$ such that $\omega^\infty : \mathcal{SH}(k)^{\mathrm{eff}} \rightarrow \mathcal{SH}^{S^1}(k)$ is t -exact.
- (5) Verify that $\mathrm{Span}(\mathrm{Fin})_\bullet$ is a complete Segal space.
- (4) Verify that the canonical functor $\mathrm{Fin} \rightarrow \mathrm{Span}(\mathrm{Fin})$ preserves finite coproducts.
- (5) Let \mathcal{C} be a symmetric monoidal ∞ -category and $A \in \mathrm{CAlg}(\mathcal{C})$. Show that the multiplication $m : A \otimes A \rightarrow A$ is associative and unital up to homotopy.
- (6) Let \mathcal{C} be a symmetric monoidal ∞ -category. Show that the tensor product on \mathcal{C} satisfies the pentagon axiom. (I.e., given $X, Y, Z, W \in \mathcal{C}$, the two evident equivalences $X \otimes (Y \otimes (Z \otimes W)) \simeq ((X \otimes Y) \otimes Z) \otimes W$ are homotopic.)
- (7) Verify that the localizations $\mathcal{P}(\mathrm{Sm}_S) \rightarrow \mathcal{Spc}(S)$ and $\mathcal{P}(\mathrm{Sm}_S)_* \rightarrow \mathcal{Spc}(S)_*$ are symmetric monoidal.
- (8) Let \mathcal{C} be a presentably symmetric monoidal ∞ -category, $X \in \mathcal{C}$ and $W \subset \mathrm{Ar}(\mathcal{C})$ a strongly saturated class of morphisms corresponding to a symmetric monoidal localization. Show that $(L_W \mathcal{C})[(L_W X)^{-1}] \simeq L_{W[X^{-1}]}(\mathcal{C}[X^{-1}])$, for a class of morphisms $W[X^{-1}]$ that you should describe.