(1) Show that for $X \in \mathrm{Sm}_S$, the projection $X \times \mathbb{A}^1 \to X$ is a naive $\mathbb{A}^1$-homotopy equivalence.
(2) Let k be the spectrum of a field and consider $\mathbb{A}^1 \setminus 0 \in \mathcal{P}(\mathrm{Sm}_k)$. Show that this presheaf is motivically local.
(3) In the category $\mathcal{S}\mathrm{pc}(S)$, show that the suspension of $\mathbb{A}^1 \setminus 0$ is given by $\mathbb{P}^1$.
(4) Let $U \to V \in \mathrm{Sm}_S$ be an étale morphism. Let $Z \subset U$ be closed (not necessarily smooth), mapping isomorphically to its image in V, also assumed closed. Show that $U/U \setminus Z \to V/V \setminus Z$ becomes an equivalence in $\mathcal{S}\mathrm{pc}(S)$. What does this have to do with "excision"?
(5) Show that $L_{\mathbb{A}^1}\mathcal{P}(\mathrm{Sm}_S)$ is equivalent to $\mathcal{P}(\mathcal{C})$, for some ∞-category $\mathcal{C}$ that you should describe.
(6) Consider the adjunction

$$\mathcal{P}(\Delta) \rightleftarrows \mathcal{S}\mathrm{pc}$$

(where the left adjoint is left Kan extension along $\Delta \to *$). Show that the right adjoint is fully faithful. Deduce that the Bousfield localization of $\mathcal{P}(\Delta)$ at the (strongly saturated class generated by the) maps $\{[n] \to [0] \mid n\}$ is $\mathcal{S}\mathrm{pc}$.
(7) Let $\mathcal{C}$ be a small category and $\mathcal{D}$ be a presentable category. Set $\mathcal{P}_{\mathcal{D}}(\mathcal{C}) = \mathrm{Fun}(\mathcal{C}^{op}, \mathcal{D})$. Using the tensoring of $\mathcal{D}$ over spaces, construct a tensoring $\mathcal{P}(\mathcal{C}) \times \mathcal{P}_{\mathcal{D}}(\mathcal{C}) \to \mathcal{P}_{\mathcal{D}}(\mathcal{C})$. Prove that, for $c \in \mathcal{C}$, $d \in \mathcal{D}$ and $F \in \mathcal{P}_{\mathcal{D}}(\mathcal{C})$ we have

$$\mathrm{Map}_{\mathcal{P}_{\mathcal{D}}(\mathcal{C})}(c \times d, F) \simeq \mathrm{Map}_{\mathcal{D}}(d, F(c)).$$

[*Hint:* you may wish to consider first the case $\mathcal{D} = \mathcal{P}(\mathcal{E})$.]