

- (1) Show that for  $X \in \text{Sm}_S$ , the projection  $X \times \mathbb{A}^1 \rightarrow X$  is a naive  $\mathbb{A}^1$ -homotopy equivalence.
- (2) Let  $k$  be the spectrum of a field and consider  $\mathbb{A}^1 \setminus 0 \in \mathcal{P}(\text{Sm}_k)$ . Show that this presheaf is motivically local.
- (3) In the category  $\mathcal{Spc}(S)$ , show that the suspension of  $\mathbb{A}^1 \setminus 0$  is given by  $\mathbb{P}^1$ .
- (4) Let  $U \rightarrow V \in \text{Sm}_S$  be an étale morphism. Let  $Z \subset U$  be closed (not necessarily smooth), mapping isomorphically to its image in  $V$ , also assumed closed. Show that  $U/U \setminus Z \rightarrow V/V \setminus Z$  becomes an equivalence in  $\mathcal{Spc}(S)$ . What does this have to do with “excision”?
- (5) Show that  $L_{\mathbb{A}^1} \mathcal{P}(\text{Sm}_S)$  is equivalent to  $\mathcal{P}(\mathcal{C})$ , for some  $\infty$ -category  $\mathcal{C}$  that you should describe.
- (6) Consider the adjunction

$$\mathcal{P}(\Delta) \rightleftarrows \mathcal{Spc}$$

(where the left adjoint is left Kan extension along  $\Delta \rightarrow *$ ). Show that the right adjoint is fully faithful. Deduce that the Bousfield localization of  $\mathcal{P}(\Delta)$  at the (strongly saturated class generated by the) maps  $\{[n] \rightarrow [1] \mid n\}$  is  $\mathcal{Spc}$ .

- (7) Let  $\mathcal{C}$  be a small category and  $\mathcal{D}$  be a presentable category. Set  $\mathcal{P}_{\mathcal{D}}(\mathcal{C}) = \text{Fun}(\mathcal{C}^{op}, \mathcal{D})$ . Using the tensoring of  $\mathcal{D}$  over spaces, construct a tensoring  $\mathcal{P}(\mathcal{C}) \times \mathcal{P}_{\mathcal{D}}(\mathcal{C}) \rightarrow \mathcal{P}_{\mathcal{D}}(\mathcal{C})$ . Prove that, for  $c \in \mathcal{C}$ ,  $d \in \mathcal{D}$  and  $F \in \mathcal{P}_{\mathcal{D}}(\mathcal{C})$  we have

$$\text{Map}_{\mathcal{P}_{\mathcal{D}}(\mathcal{C})}(c \times d, F) \simeq \text{Map}_{\mathcal{D}}(d, F(c)).$$

[*Hint:* you may wish to consider first the case  $\mathcal{D} = \mathcal{P}(\mathcal{E})$ .]