(1) Let $\mathcal{C}$ be a small ∞-category with a Grothendieck topology and $X \in \mathcal{C}$. Make sense of and prove the following:

$$\mathcal{S}\mathrm{hv}(\mathcal{C}_{/X}) \simeq \mathcal{S}\mathrm{hv}(\mathcal{C})_{/X}.$$

(2) Verify that the family of Nisnevich covering sieves satisfies the axioms of a Grothendieck topology.

(3) Let X be any scheme. Show that $\mathcal{S}\mathrm{hv}_{\mathrm{Nis}}(\mathcal{E}\mathrm{t}_X^{fp}) \simeq \mathcal{S}\mathrm{hv}_{\mathrm{Nis}}(\mathcal{E}\mathrm{t}_X)$. (The Nisnevich topology on both sides is defined in terms of possibly infinite covering families.) We denote these two equivalent ∞-topoi by X_{Nis}.

(4) If X is qcqs, show that X_{Nis} is also equivalent to the localization of $\mathcal{P}(\mathcal{E}\mathrm{t}_X^{fp})$ at the sieves corresponding to finite covering families.

(5) Let X_α be a cofiltered system of qcqs schemes with affine transition maps. Write $X = \lim_\alpha X_\alpha$ and let $Y_1, \ldots, Y_n \to X$ be a finitely presented Nisnevich cover. Show that there exists an index α_0 and a finitely presented Nisnevich cover $\tilde{Y}_1, \ldots, \tilde{Y}_n \to X_{\alpha_0}$ such that $Y_i \simeq \tilde{Y}_i \times_{X_{\alpha_0}} X$.

(6) Let X_α as above. Show that $X_{\mathrm{Nis}} \simeq \lim_\alpha (X_\alpha)_{\mathrm{Nis}}$. [*Hint:* view this as a colimit in Pr^L. First establish the analogous claim for $\mathcal{E}\mathrm{t}_{(-)}^{fp}$.]