(1) Show that the category of groupoid objects in sets (a subcategory of $\mathrm{Fun}(\Delta^{\mathrm{op}}, Set)$) is equivalent to the usual category of groupoids.

(2) Show that $f : X \to Y \in \mathcal{S}\mathrm{pc}$ is an epimorphism if and only if $\pi_0(f) : \pi_0(X) \to \pi_0(Y)$ is a surjection.

(3) Let $F \xrightarrow{i} X \to Y \in \mathcal{S}\mathrm{pc}_*$ be a fiber sequence. Construct an equivalence

$$fib(cof(i) \to Y) \simeq \Sigma\Omega Y \wedge F.$$

[*Hint:* you may wish to recall or prove that for pointed spaces A, B the pushout of $A \leftarrow A \times B \to B$ is $\Sigma A \wedge B$.]

(4) Let $\mathcal{C}$ be a small ∞-category and $f_i : X_i \to X \in \mathcal{C}$ some collection of maps. Describe the Čech nerve of the map

$$\coprod_i y(X_i) \to y(X) \in \mathcal{P}(\mathcal{C})$$

as a subobject of X.

(5) For $X \in \mathcal{S}\mathrm{pc}$, construct an equivalence

$$\mathcal{S}\mathrm{pc}_{/X} \simeq \mathrm{Fun}(X, \mathcal{S}\mathrm{pc}).$$

[*Hint:* view both sides as a functor of X.] Under this equivalence, describe $\underline{\pi}_1(X) \in \mathrm{Fun}(X, Set)$.

(6) Show that $\mathcal{S}\mathrm{pc}_{/S^1}$ is equivalent to the category of pairs (X, η) where $X \in \mathcal{S}\mathrm{pc}$ and $\eta : X \xrightarrow{\simeq} X$ is an autoequivalence. Prove that $\mathcal{S}\mathrm{pc}_{/S^1}$ has homotopy dimension 1.

(7) Let $\mathcal{P}$ be a property of morphisms in an ∞-topos $\mathcal{X}$. We say the property is *local* if whenever given a cartesian square

$$\begin{array}{ccc} X & \longrightarrow & Y \\ {\scriptstyle f'}\downarrow & & \downarrow{\scriptstyle f} \\ Z & \xrightarrow{p} & W \end{array}$$

with p an epimorphism and $f' \in \mathcal{P}$, also $f \in \mathcal{P}$. Show that equivalences are local.

(8) Show that epimorphisms are local. More generally, show that n-truncated morphisms are local for every n.

(9) Let $X \to Y$ be an n-connective morphism in an ∞-topos $\mathcal{X}$ of homotopy dimension $\leq d$. Show that $\mathrm{Map}(*, X) \to \mathrm{Map}(*, Y)$ is $(n-d)$-connective.

(10) Learn about an example of a non-hypercomplete ∞-topos.